CALCULATION OF CONVECTIVE STREAMS IN THE LIQUID CORE OF SOLIDIFYING BODIES OF VERY SIMPLE SHAPE

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Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 1, pp. 92–100, 1966

UDC 536.248

An analytical solution is given to the problem of distribution of velocity, temperature and heat flux in the liquid core of solidifying bodies of very simple shape, with natural convection and an arbitrary law of motion of the two-phase boundary.

The solution is well known for the problem of cooling of the liquid core of solidifying bodies of simple shape, when there is no mixing of the melt [1]. In the solidification of ordinary ingots and castings with a superheated liquid core, convective flows occur due to variation of density of the metal with temperature.

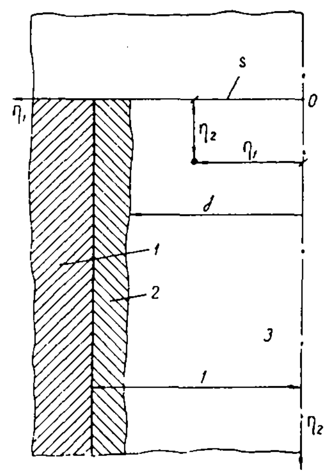

Fig. 1. Location of the semi-infinite slab or cylinder in the coordinate system: S is the surface on which there are no heat fluxes along the axis η_2; 1) mold; 2) solidified crust; 3) liquid melt.

The present paper gives a solution to the problem of cooling of the liquid core of a solidifying semi-infinite slab and a semi-infinite cylinder, taking account of convective flow. The thermal and hydrodynamic process of convective displacement of melt is an unsteady one in the case under examination. Solutions are not known for problems of this kind.

We arrange the slab and cylinder so that the thermal plane (or respectively the axis) is parallel to the gravity vector. We assume that there is no heat exchange over the top face (Fig. 1). This case corresponds approximately to casting of an ingot in a heated mold. It may also be assumed that the pressure along the axis η_2 is constant. Conditions for heat release are symmetrical at the body boundaries. Then the problem for a slab and a cylinder reduces to solution of a system of three equations:

the equation of convective heat transfer

$$\frac{\partial t}{\partial \mathrm{Fo}} + \mathrm{Pr}\left(\mathrm{Re}_1 \frac{\partial t}{\partial \eta_1} + \mathrm{Re}_2 \frac{\partial t}{\partial \eta_2}\right) = \frac{\partial^2 t}{\partial \eta_1^2} + \frac{m}{\eta_1}\frac{\partial t}{\partial \eta_1} + \frac{\partial^2 t}{\partial \eta_2^2}, \tag{1}$$

the equation of motion

$$\frac{1}{\mathrm{Pr}}\frac{\partial \mathrm{Re}_2}{\partial \mathrm{Fo}} + \mathrm{Re}_1 \frac{\partial \mathrm{Re}_2}{\partial \eta_1} + \mathrm{Re}_2 \frac{\partial \mathrm{Re}_2}{\partial \eta_2} = \frac{\partial^2 \mathrm{Re}_2}{\partial \eta_1^2} + \frac{m}{\eta_1}\frac{\partial \mathrm{Re}_2}{\partial \eta_1} + \frac{\partial^2 \mathrm{Re}_2}{\partial \eta_2^2} + \mathrm{Gr}\, t \tag{2}$$

and the continuity equation

$$\frac{\partial \mathrm{Re}_1}{\partial \eta_1} + \frac{m}{\eta_1}\mathrm{Re}_1 + \frac{\partial \mathrm{Re}_2}{\partial \eta_2} = 0. \tag{3}$$

It may be assumed that the width of the liquid zone varies only as a function of time and is a given quantity:

$$j = j(\mathrm{Fo}). \tag{4}$$

It has been shown [2] that, depending on the law of solidification, if the heat flux from the liquid core to the crust of the ingot is known, the conditions on the inner surface which satisfy the given law of solidification may be determined.

At the solidification front the relative superheat of the melt is zero:

$$t|_{\eta_1=j} = 0. \tag{5}$$

The dimensionless velocities are also zero at this front:

$$\mathrm{Re}_1|_{\eta_1=j} = 0, \tag{6}$$

$$\mathrm{Re}_2|_{\eta_1=j} = 0. \tag{7}$$

It follows from the symmetry of the bodies in question that

$$\left.\frac{\partial t}{\partial \eta_1}\right|_{\eta_1=0} = 0, \tag{8}$$

$$\mathrm{Re}_1|_{\eta_1=0} = 0, \tag{9}$$

$$\left.\frac{\partial \mathrm{Re}_2}{\partial \eta_1}\right|_{\eta_1=0} = 0. \tag{10}$$

The condition for no heat exchange at the top face is

$$\left.\frac{\partial t}{\partial \eta_2}\right|_{\eta_2=0} = 0. \qquad (11)$$

The condition that the velocity fields are zero in the melt at time zero is

$$\mathrm{Re}_2|_{\mathrm{Fo}=0} = 0, \qquad (12)$$

$$\mathrm{Re}_1|_{\mathrm{Fo}=0} = 0. \qquad (12a)$$

For mixing of the melt under natural convection there cannot be displacement of the liquid core as a whole, and so the amount of cold melt descending must be equal to the amount of hot melt ascending, which leads to the condition

$$\int_0^j \mathrm{Re}_2 \eta_1^m d\eta_1 = 0. \qquad (13)$$

Finally, an initial temperature distribution should be assigned, which we take to be independent of the coordinate η_2. However, as follows from the analysis of the problem solved previously for the temperature field in an unmixed core [1], it is more convenient to assign the heat flux from the liquid melt to the crust being formed rather than the initial temperature distribution directly. For any reasonable assumptions about the nature of variation of heat flux with time, it is easy to obtain a corresponding initial temperature distribution. We therefore assign the heat flux from the liquid melt to the crust at some level or other, e.g., at $\eta_2 = 0$, and require in addition that at time zero the relative superheat does not depend on the coordinate η_2:

$$-\left.\frac{\partial t}{\partial \eta_1}\right|_{\substack{\eta_1=j\\ \eta_2=0}} = q(\mathrm{Fo}), \qquad (14)$$

$$\left.\frac{\partial t}{\partial \eta_2}\right|_{\mathrm{Fo}=0} = 0. \qquad (15)$$

These two conditions are completely equivalent to assigning an initial temperature distribution in the melt, independent of the coordinate η_2. During cooling the melt temperature, in general, depends on η_2 because of movement of hot sections of the melt upwards, and of cold sections downwards.

In order to solve the problem as formulated, we represent the functions sought in the form of the following power series with variable coefficients:

$$t = \sum_{k=0}^{\infty} \frac{C_k(\eta_2, \mathrm{Fo})}{k!} (j^2 - \eta_1^2)^k, \qquad (16)$$

$$\mathrm{Re}_2 = \sum_{k=0}^{\infty} \frac{D_k(\eta_2, \mathrm{Fo})}{k!} (j^2 - \eta_1^2)^k, \qquad (17)$$

$$\mathrm{Re}_1 = \eta_1 \sum_{k=0}^{\infty} \frac{E_k(\eta_2, \mathrm{Fo})}{k!} (j^2 - \eta_1^2)^k. \qquad (18)$$

The series (16) and (17) do not differ in essence from that used in (1), and so operations with them do not cause difficulties. It follows from (18) that

$$\frac{\partial \mathrm{Re}_1}{\partial \eta_1} = \sum_{k=0}^{\infty} \frac{E_k}{k!} (j^2 - \eta_1^2)^k - 2\eta_1^2 \sum_{k=0}^{\infty} \frac{E_{k+1}}{k!} (j^2 - \eta_1^2)^k =$$

$$= \sum_{k=0}^{\infty} \frac{E_k}{k!} (j^2 - \eta_1^2)^k + 2 \sum_{k=0}^{\infty} \frac{E_{k+1}}{k!} (j^2 - \eta_1^2)^{k+1} -$$

$$- 2j^2 \sum_{k=0}^{\infty} \frac{E_{k+1}}{k!} (j^2 - \eta_1^2)^k =$$

$$= \sum_{k=0}^{\infty} \frac{E_k - 2j^2 E_{k+1}}{k!} (j^2 - \eta_1^2)^k +$$

$$+ 2 \sum_{k=0}^{\infty} \frac{E_{k+1}(k+1)}{(k+1)!} (j^2 - \eta_1^2)^{k+1} =$$

$$= \sum_{k=0}^{\infty} \frac{E_k - 2j^2 E_{k+1}}{k!} (j^2 - \eta_1^2)^k + 2 \sum_{k=0}^{\infty} \frac{kE_k}{k!} (j^2 - \eta_1^2)^k;$$

$$\frac{\partial \mathrm{Re}_1}{\partial \eta_1} = \sum_{k=0}^{\infty} \frac{(1+2k)E_k - 2j^2 E_{k+1}}{k!} (j^2 - \eta_1^2)^k. \qquad (19)$$

We illustrate the manner of obtaining the product of the two independent infinite series with the example

$$\mathrm{Re}_2 \frac{\partial t}{\partial \eta_2} = \sum_{i=0}^{\infty} \frac{D_i}{i!} (j^2 - \eta_1^2)^i \sum_{l=0}^{\infty} \frac{\frac{\partial}{\partial \eta_2} C_l}{l!} (j^2 - \eta_1^2)^l.$$

Here the summation indices are denoted by different symbols, to underline the independence of one from another. Let us designate: $i + l = k$, when $l = k - i$. It is evident that l cannot be negative, and therefore i cannot be greater than k. Then

$$\mathrm{Re}_2 \frac{\partial t}{\partial \eta_2} =$$

$$= \sum_{k=0}^{\infty} \sum_{i=0}^{k} \frac{D_i}{i!} \frac{\partial}{\partial \eta_2} C_{k-i} [(k-i)!]^{-1} (j^2 - \eta_1^2)^k. \qquad (20)$$

In a similar way, we obtain the products

$$\mathrm{Re}_1 \frac{\partial t}{\partial \eta_1}, \quad \mathrm{Re}_1 \frac{\partial \mathrm{Re}_2}{\partial \eta_1} \quad \text{and} \quad \mathrm{Re}_2 \frac{\partial \mathrm{Re}_2}{\partial \eta_2}.$$

Substituting expressions (16)–(18) into (1)–(3), and equating coefficients with like powers $j^2 - \eta_1^2$ to zero, we obtain the following recurrence relations for coefficients C_k, D_k, E_k:

$$C_{k+2} = \frac{1}{4j^2} \{ 2(m + 1 + 2k) + jj') C_{k+1} +$$

$$+\frac{\partial}{\partial \mathrm{Fo}} C_k - \frac{\partial^2}{\partial \eta_2^2} C_k +$$

$$+ 2\,\mathrm{Pr}\, k! \sum_{i=0}^{k} \frac{[(k-i)C_{k-i} - j^2 C_{k-i+1}]E_i}{(k-i)!\, i!} +$$

$$+ \mathrm{Pr}\, k! \sum_{i=0}^{k} D_i \frac{\partial}{\partial \eta_2} C_{k-i} [i!(k-i)!]^{-1}, \tag{21}$$

$$D_{k+2} = \frac{1}{4j^2}\Big\{ 2(m+1+2k+jj')D_{k+1} +$$

$$+ \frac{1}{\mathrm{Pr}} \frac{\partial}{\partial \mathrm{Fo}} D_k - \frac{\partial^2}{\partial \eta_2^2} D_k - \mathrm{Gr}\, C_k +$$

$$+ 2k! \sum_{i=0}^{k} \frac{[(k-i)D_{k-i} - j^2 D_{k-i+1}]E_i}{(k-i)!\, i!} +$$

$$+ k! \sum_{i=0}^{k} D_i \frac{\partial}{\partial \eta_2} D_{k-i} [i!(k-i)!]^{-1}, \tag{22}$$

$$E_{k+1} = \frac{1}{2j^2}\left\{ (m+1+2k)E_k + \frac{\partial}{\partial \eta_2} D_k \right\}. \tag{23}$$

From the boundary conditions (5)–(7) we have at once

$$C_0 = 0, \tag{24}$$

$$D_0 = 0, \tag{25}$$

$$E_0 = 0. \tag{26}$$

Functions C_1 and D_1 must be determined from the remaining boundary conditions. Function E_1 is determined from recurrence relation (23):

$$E_1 = 0. \tag{27}$$

Performing a calculation according to (21)–(23), we obtain

$$C_2 = \frac{m-1+jj'}{2j^2} C_1, \tag{28}$$

$$C_3 = \frac{(m+1+jj')(m+3+jj')}{4j^4} C_1 +$$

$$+ \frac{1}{4j^2} \frac{\partial}{\partial \mathrm{Fo}} C_1 - \frac{1}{4j^2} \frac{\partial^2}{\partial \eta_2^2} C_1, \tag{29}$$

$$D_2 = \frac{m+1+jj'}{2j^2} D_1, \tag{30}$$

$$D_3 = \frac{(m+1+jj')(m+3+jj')}{4j^4} D_1 +$$

$$+ \frac{1}{4\mathrm{Pr}\, j^2} \frac{\partial}{\partial \mathrm{Fo}} D_1 - \frac{1}{4j^2} \frac{\partial^2}{\partial \eta_2^2} D_1 - \frac{1}{4j^2} \mathrm{Gr}\, C_1, \tag{31}$$

$$E_2 = \frac{1}{2j^2} \frac{\partial}{\partial \eta_2} D_1, \tag{32}$$

$$E_3 = \frac{m+6+jj'}{4j^4} \frac{\partial}{\partial \eta_2} D_1. \tag{33}$$

Substituting (16) in place of t, we have, from condition (14),

$$C_1(0, \mathrm{Fo}) = \frac{1}{2j} q. \tag{34}$$

From (11), in the same way, we obtain

$$\left.\frac{\partial C_k}{\partial \eta_2}\right|_{\eta_2=0} = 0. \tag{35}$$

When k = 1

$$\frac{\partial}{\partial \eta_2} C_1(0, \mathrm{Fo}) = 0. \tag{36}$$

When k = 2 we also obtain (36).
When k = 3

$$\frac{\partial}{\partial \eta_2} C_3(0, \mathrm{Fo}) = 0.$$

Using expression (29) in place of C_3, we obtain

$$\left.\left(\frac{\partial^2 C_1}{\partial \mathrm{Fo}\, \partial \eta_2} - \frac{\partial^3 C_1}{\partial \eta_2^3} \right)\right|_{\eta_2=0} = 0.$$

After integration we have

$$\left.\left(\frac{\partial C_1}{\partial \mathrm{Fo}} - \frac{\partial^2 C_1}{\partial \eta_2^2} + \Phi \right)\right|_{\eta_2=0} = 0.$$

Since the integration was carried out with respect to the variable η_2, the arbitrary constant of integration may depend on Fo. Therefore

$$\frac{\partial^2}{\partial \eta_2^2} C_1(0, \mathrm{Fo}) = \Phi(\mathrm{Fo}) + \frac{\partial}{\partial \mathrm{Fo}} C_1(0, \mathrm{Fo}). \tag{37}$$

Since we require condition (35) to hold for coefficients C_k with indices k > 3, we may obtain all the successive coefficients of the expansion of function $C_1(\eta_2, \mathrm{Fo})$ as a series in powers of η_2. In the first approximation, confining ourselves to the coefficients found, we obtain

$$C_1(\eta_2, \mathrm{Fo}) = C_1(0, \mathrm{Fo}) +$$

$$+ \frac{1}{2}\left[\Phi + \frac{\partial}{\partial \mathrm{Fo}} C_1(0, \mathrm{Fo}) \right] \eta_2^2. \tag{38}$$

To determine function Φ we use (15), when

$$\left.\frac{\partial C_k}{\partial \eta_2}\right|_{\mathrm{Fo}=0} = 0.$$

When k = 1 we have

$$\left.\frac{\partial C_1}{\partial \eta_2}\right|_{\mathrm{Fo}=0} = 0.$$

Substituting expression (38) in place of C_1, we obtain

$$\Phi(0) = -\frac{\partial}{\partial \mathrm{Fo}} C_1(0, 0). \tag{39}$$

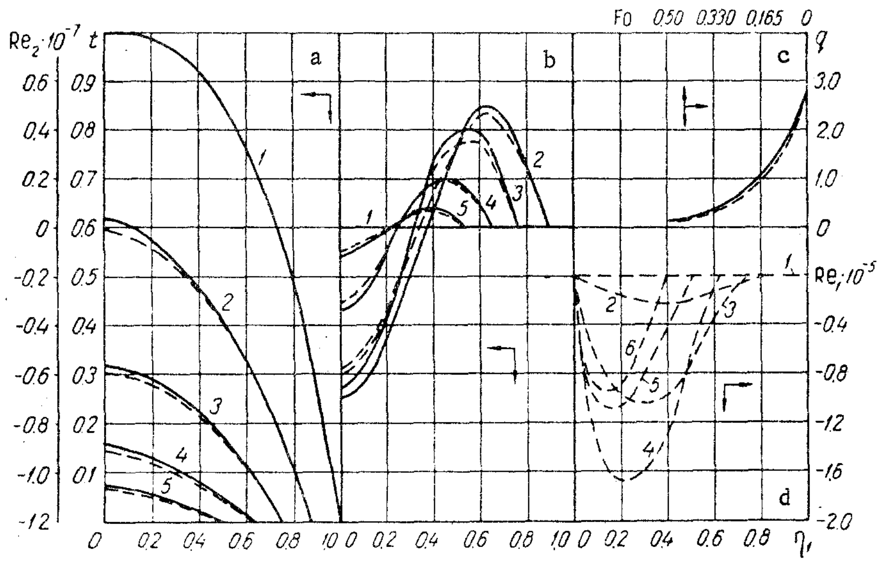

Fig. 2. Distribution of temperature, heat flux, and velocity in a semi-infinite slab undergoing solidification: a) curves of relative superheat temperature t; b) vertical component of dimensionless velocity Re_2; c) horizontal component of dimensionless velocity Re_1; d) dimensionless heat flux from melt to crust q; 1) Fo = 0; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.4; 6) 0.5.

When k = 2 we obtain the same condition. When k = 3, replacing C_3 by C according to (29), and expressing C_1 according to (38), we obtain

$$\frac{\partial}{\partial \mathrm{Fo}} \Phi(0) = -\frac{\partial^2}{\partial \mathrm{Fo}^2} C_1(0, 0). \tag{40}$$

Continuing this process when k > 3, we obtain further terms of the expansion of function $\Phi(\mathrm{Fo})$. In the first approximation

$$\Phi(\mathrm{Fo}) = -\left[\frac{\partial}{\partial \mathrm{Fo}} C_1(0, \mathrm{Fo})\right]\Big|_{\mathrm{Fo}=0} - \mathrm{Fo}\left[\frac{\partial^2}{\partial \mathrm{Fo}^2} C_1(0, \mathrm{Fo})\right]\Big|_{\mathrm{Fo}=0}. \tag{41}$$

Indeed, with the aid of (14), (11), and (15), function $C_1(\eta_2, \mathrm{Fo})$ is completely determined (expressions (34), (38), and (41) determine it in the first approximation).

In order to determine function D_1, we must use the remaining conditions (12) and (13).

The validity of the following relations may be shown:

$$\sum_{l=0}^{\infty} \frac{D_l}{l!} \int (j^2 - \eta_1^2)^l \, d\eta_1 = - \sum_{l=0}^{\infty} \sum_{i=0}^{\infty} \frac{D_{l+i}(l+i)!}{l!(l+2i+1)!} (j^2 - \eta_1^2)^l (j - \eta_1)^{2i+1}, \tag{42}$$

$$\sum_{l=0}^{\infty} \frac{D_l}{l!} \int (j^2 - \eta_1^2)^l \eta_1 \, d\eta_1 = -\frac{1}{2} \sum_{l=0}^{\infty} \frac{D_l}{(l+1)!} (j^2 - \eta_1^2)^{l+1}. \tag{43}$$

Using the first of these, we write (13) for the slab (m = 0) in the form

$$\int_0^j \mathrm{Re}_2 \, d\eta_1 - \sum_{l=0}^{\infty} \sum_{i=0}^{\infty} \frac{D_{l+i}(l+i)!}{l!(l+2i+1)!} j^{2(l+i)+1} = 0.$$

Putting $l + i = k$, $l = k - i$, we obtain

$$\int_0^j \mathrm{Re}_2 \, d\eta_1 - \sum_{k=0}^{\infty} \sum_{i=0}^{k} \frac{D_k k!}{(k-i)!(k+i+1)!} j^{2k+1} = 0. \tag{44}$$

Restricting ourselves to the first few terms of the series, we obtain from (44)

$$0.67 D_1 + 0.267 D_2 j^2 + 0.073 D_3 j^4 + \ldots = 0. \tag{45}$$

Substituting their expressions in terms of D_1 instead of D_2 and D_3, we obtain when m = 0 from (30) and (31), after simple transformations,

$$\frac{\partial}{\partial \mathrm{Fo}} D_1 + \frac{\mathrm{Pr}}{j^2}(47.5 + 11.4 jj') D_1 = \mathrm{Pr}\,\mathrm{Gr}\, C_1 + \mathrm{Pr} \frac{\partial^2 D_1}{\partial \eta_2^2}. \tag{46}$$

Here we have dropped the term containing jj' as a square in the brackets, in order to effect some simplification, it being negligible in comparison with the terms retained.

The solution of the equation formulated has the form

$$D_1 = \mathrm{Pr}\,\mathrm{Gr} \exp\left(-\mathrm{Pr}\int_0^{\mathrm{Fo}} \frac{47.5 + 11.4 jj'}{j^2} d\mathrm{Fo}\right) \times \int_0^{\mathrm{Fo}} (C_1 + f) \exp\left(\mathrm{Pr}\int_0^{\mathrm{Fo}} \frac{47.5 + 11.4 jj'}{j^2} d\mathrm{Fo}\right) d\mathrm{Fo}. \tag{47}$$

The function f is determined by the expression

$$f = \mathrm{Pr} \exp\left(-\mathrm{Pr}\int_0^{\mathrm{Fo}} \frac{47.5 + 11.4 jj'}{j^2} d\mathrm{Fo}\right) \times \int_0^{\mathrm{Fo}} \frac{\partial^2 C_1}{\partial \eta_2^2} \exp\left(\mathrm{Pr}\int_0^{\mathrm{Fo}} \frac{47.5 + 11.4 jj'}{j^2} d\mathrm{Fo}\right) d\mathrm{Fo}. \tag{48}$$

It follows from (12) that

$$D_k|_{\mathrm{Fo}=0} = 0. \tag{49}$$

It is easy to see from (47) that this condition is satisfied when k = 1 and k = 2. When k = 3, expressing D_3 in terms of D_1 according to (31) and performing the differentiation, we obtain

$$\frac{1}{\mathrm{Pr}} \frac{\partial}{\partial \mathrm{Fo}} D_1|_{\mathrm{Fo}=0} = \frac{1}{\mathrm{Pr}} \mathrm{Pr}\,\mathrm{Gr}(C_1 + f)|_{\mathrm{Fo}=0}; \quad D_3|_{\mathrm{Fo}=0} = 0.$$

Thus, even when k = 3, (12) is satisfied identically. Condition (12a) is also satisfied, which may easily be verified from the recurrence formula (23).

Condition (13) for the cylinder (m = 1) may be written, using (43), in the form

$$\int_0^j \mathrm{Re}_2 \eta_1 \, d\eta_1 - \frac{1}{2} \sum_{k=0}^{\infty} \frac{D_k}{(k+1)!} j^{2k+2} = 0. \tag{50}$$

In this expression the summation index l has been replaced by k. Restricting ourselves as before to the first few terms of the series, we obtain, from (50),

$$0.50 D_1 + 0.167 D_2 j^2 + 0.0417 D_3 j^4 + \ldots = 0.$$

After transformations similar to those carried out previously, we have

$$\frac{\partial}{\partial \mathrm{Fo}} D_1 + \frac{\mathrm{Pr}}{j^2}(144 + 68 jj') D_1 = \mathrm{Pr}\,\mathrm{Gr}\, C_1 + \mathrm{Pr} \frac{\partial^2 D_1}{\partial \eta_2^2}.$$

The equation obtained is analogous to (46), and therefore its solution has the analogous form:

$$D_1 = \mathrm{Pr}\,\mathrm{Gr} \exp\left(-\mathrm{Pr}\int_0^{\mathrm{Fo}} \frac{144 + 68 jj'}{j^2} d\mathrm{Fo}\right) \times \int_0^{\mathrm{Fo}} (C_1 + f) \exp\left(\mathrm{Pr}\int_0^{\mathrm{Fo}} \frac{144 + 68 jj'}{j^2} d\mathrm{Fo}\right) d\mathrm{Fo};$$

$$f = \Pr\exp\left(-\Pr\int_0^{\mathrm{Fo}} \frac{144+68jj'}{j^2}\, d\mathrm{Fo}\right) \times$$
$$\times \int_0^{\mathrm{Fo}} \frac{\partial^2 C_1}{\partial \eta_2^2} \exp\left(\Pr\int_0^{\mathrm{Fo}} \frac{144+68jj'}{j^2}\, d\mathrm{Fo}\right) d\mathrm{Fo}.$$

Here, as in the previous case, conditions (12) and (12a) are satisfied identically.

As an illustration a calculation has been done of distributions of temperature, heat flux, and velocity in the liquid core of a semi-infinite slab of thickness $2R = 0.4$ m, with a linear law of crust growth (Fig. 2):

$$j = 1 - 1.2\,\mathrm{Fo}.$$

The liquid melt is steel; $a = 0.555 \cdot 10^{-5}$ m²/sec; $\nu = 0.36 \cdot 10^{-6}$ m²/sec; $\lambda = 23$ W/m · degree; $\beta = 25 \times \times 10^{-5}$ l/degree; $T_K = 1500°$ C; the maximum initial temperature $T_0 = 1525°$ C; Pr = 0.065; Gr = $3.47 \cdot 10^9$. The dimensionless heat flux is given in the form

$$q = 2Lj\exp(-5\,\mathrm{Fo}).$$

The coefficient of proportionality (a constant) L was determined in the last stage of calculation from the condition $t_{\max}|_{\mathrm{Fo}=0} = 1$, and proved to be 1.3. The calculation was done at $\eta_2 = 0$ and $\eta_2 = 0.1$.

NOTATION

$t = (T - T_K)/(T_0 - T_K)$ —relative superheat temperature of melt; T—temperature of melt; T_K—crystallization temperature; T_0—maximum initial temperature of melt; $\mathrm{Fo} = a\tau/R^2$—dimensionless time; a—thermal diffusivity of melt; τ—time; R—half-thickness of slab, or radius of cylinder; $\Pr = \nu/a$—Prandtl number; ν—kinematic viscosity of melt; $\eta_i = x_i/R$—dimensionless coordinate; j—dimensionless distance from axis of slab or cylinder to solidification front; $\mathrm{Re}_i = u_iR/\nu$—dimensionless component of melt velocity along coordinate η_i; m—shape factor equal to 0 for slab, and 1 for cylinder; $\mathrm{Gr} = g\beta(T_0 - T_K)R^3/\nu^2$ —Grashof number; g—acceleration due to gravity; β—coefficient of volume expansion of melt; $q = QR/\lambda(T_0 - T_K)$ —dimensionless heat flux from liquid core to crust of ingot; Q—heat flux to ingot crust; λ—thermal conductivity (molecular) of melt.

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